

# Classical self-energy and anomaly

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We study the problem of self-energy of pointlike charges in higher dimensional static spacetimes. Their energy, as a functional of the spacetime metric, is invariant under a specific continuous transformation of the metric. We show that the procedure of regularization of this formally divergent functional breaks this symmetry and results in an anomalous contribution to the finite renormalized self-energy. We proposed a method of calculation of this anomaly and presented an explicit expressions for it in the case of a scalar charge in four and five-dimensional static spacetimes. This anomalous correction proves to be zero in even dimensions, but it does not vanish in odd-dimensional spacetimes.

## I. INTRODUCTION

The problem of calculation of the self-energy of charged particles has a long history going to classical works [1–6] and many others, where the electromagnetic origin of the electron mass had been studied. Achievements of renormalization techniques in quantum field theory contributed a lot to the understanding of the problem. Classical self-energy of an electron can be derived as the limit of its quantum value. It had been shown [7, 8], that there exists a correct quantum-to-classical correspondence for the self-energy of the electron (see also [9].)

In quantum electrodynamics the self-energy of an electron diverges and, hence, should be regularized and renormalized. A classical self-energy of pointlike charges suffers similar divergences. The simplest way to regularize the energy of a classical charged particle is to smear the charge distribution. If a size  $\varepsilon$  of the smearing tends to zero, its energy diverges as  $\varepsilon^{-1}$ , so that after subtraction of this leading term in the expansion over  $\varepsilon$ , one obtains a finite expression for the self energy. The problem is that the result depends on the details of smearing.

Using methods similar to those adopted in quantum field theory allows one to formulate the renormalization procedure for a classical charged particle in a more general and efficient form. This occurs because when the size  $\varepsilon$  of a charge becomes smaller than the corresponding invariant cut-off length, the details of the charge distribution become unimportant. It should be emphasized that in the higher dimensional theories, which are widely discussed in modern physics, the classical self-energy divergence is much stronger  $\sim \varepsilon^{-(D-3)}$ , where  $D$  is the number of spacetime dimensions. As a result the problem of dependence of the self-energy on the details of the charge distribution becomes more involved. However, the covariant renormalization approach adapted from the

quantum field theory efficiently cures this ‘decease’ in any number of dimensions (see e.g. [10]).

In the presence of an external gravitational field the self-energy of a charged classical particle depends on its position. As a result there may exist a non-trivial additional force acting on the particle. This effect was discussed in detail for a special case when a charged particle is located near a static 4-dimensional black hole (see e.g. [11–17]).

In our recent paper [10] we obtained the expression for the gravitational shift of the renormalized proper mass of a classical charged particle in the higher-dimensional Majumdar-Papapetrou spacetimes. We demonstrated how the calculation of the self-energy of a pointlike static scalar charge can be reduced to the calculation of the regularized Green function of a particular operator defined in the spatial section of static spacetime.

We propose to use standard renormalization techniques of quantum field theory to single out UV divergences. For the problem in question the point-splitting regularization is particularly convenient, though one can expect that the other methods like  $\zeta$ -function regularization, dimensional regularization, and others would lead to the same finite answer (see e.g. [18]). In the limit of pointlike charges after subtraction of divergences the dependence of the renormalized self-energy on details of internal structure of the source disappear and the predictions for the finite self-energy and self-force become universal.

The proposed approach is well adapted to higher dimensions and leads to unambiguous universal predictions. In our previous paper [10] we derived the exact formula for the self-energy of static scalar charges in higher-dimensional Majumdar-Papapetrou spacetime [19], which describes a set of extremally charged black holes in equilibrium. An unexpected property of the obtained result is that dimensionality matters. In odd-dimensional spacetimes there appears an anomalous contribution to the classical self-energy of charges which depends on the space curvature. For even spacetime dimensions this anomaly of the self-energy vanishes, just

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like trace anomaly vanishes for conformal fields in odd dimensions. The difference is that the trace anomaly is a quantum effect, while the self-energy is classical.

The anomaly is a violation of some symmetry. In our case the classical self-energy of a charge distribution, before taking the limit of pointlike charge and renormalization of divergences, remains invariant under continuous symmetry transformations of the metric and the field. These local transformations involve simultaneous multiplication of the time component  $g_{tt}$  of the metric on a scalar function of spatial coordinates and conformal transformation of the spatial part  $g_{ab}$  of the space-time metric. In the next section we establish the symmetry of the energy functional under such transformation. The self-energy of pointlike charges formally diverges. Its regularization and renormalization does not respect this symmetry that results in the appearance of local anomalous contributions.

## II. SELF-ENERGY OF A SCALAR CHARGE IN A STATIC SPACETIME

Let us consider a minimally coupled massless scalar field  $\Phi$  in a static  $D$ -dimensional spacetime with the metric  $g_{\mu\nu}$

$$ds^2 = -\alpha^2 dt^2 + g_{ab} dx^a dx^b. \quad (2.1)$$

We assume that the spacetime is static  $\partial_t \alpha = \partial_t g_{ab} = 0$ . The action for this field is

$$I = -\frac{1}{8\pi} \int d^D y \sqrt{-g} \Phi^{;\mu} \Phi_{;\mu}. \quad (2.2)$$

Here

$$g = \det g_{\mu\nu} = -\alpha^2 g, \quad g = \det g_{ab}. \quad (2.3)$$

The field obeys the equation

$$\square \Phi = 0. \quad (2.4)$$

The energy  $E$  of a static configuration of fields is

$$E = \frac{1}{8\pi} \int d^{D-1} x \sqrt{g} \alpha \Phi^{;a} \Phi_{;a}. \quad (2.5)$$

It is useful to introduce another field variable  $\varphi$

$$\Phi = \alpha^{-1/2} \varphi. \quad (2.6)$$

In terms of this field the energy takes the form

$$E = \frac{1}{8\pi} \int d^{D-1} x \sqrt{g} g^{ab} \left( \varphi_{;a} - \frac{\alpha_{;a}}{2\alpha} \varphi \right) \left( \varphi_{;b} - \frac{\alpha_{;b}}{2\alpha} \varphi \right). \quad (2.7)$$

This expression for the energy formally looks like the Euclidean action of  $(D-1)$ -dimensional scalar field interacting with the external dilaton field  $\alpha$ . One can use

this analogy to reformulate the problem of calculation of the self-energy in terms of the Euclidean quantum field theory defined on  $(D-1)$ -dimensional space and in the presence of the external dilaton field.

The field  $\varphi$  satisfies the equation

$$F \varphi \equiv (\Delta + V) \varphi = 0, \quad V = \frac{(\nabla \alpha)^2}{4\alpha^2} - \frac{\Delta \alpha}{2\alpha} \equiv -\frac{\Delta(\alpha^{1/2})}{\alpha^{1/2}}. \quad (2.8)$$

Here

$$\Delta = g^{ab} \nabla_a \nabla_b. \quad (2.9)$$

Consider the following transformations of the metric Eq.(2.1) and the field  $\varphi$

$$\begin{aligned} g_{ab} &= \Omega^2 \bar{g}_{ab}, & \alpha &= \Omega^{-n} \bar{\alpha}, \\ \varphi &= \Omega^{-n/2} \bar{\varphi}, & n &= D-3. \end{aligned} \quad (2.10)$$

From the point of view of a field theory on  $(D-1)$ -dimensional spatial slice it describes simultaneous conformal transformation of the metric  $g_{ab}$  and transformation of the dilaton field  $\alpha$ . Under these transformations the energy functional Eq.(2.7) remains invariant. The operator  $F$  in Eq.(2.8) transforms homogeneously

$$F = \Omega^{-2-\frac{n}{2}} \bar{F} \Omega^{\frac{n}{2}}, \quad (2.11)$$

so that the field equation is invariant under these transformations

$$(\Delta + V) \varphi = \Omega^{-2-\frac{n}{2}} (\bar{\Delta} + \bar{V}) \bar{\varphi} = 0. \quad (2.12)$$

The energy  $E$  Eq.(2.7) is a functional of  $(D-1)$ -dimensional dynamical field  $\varphi$  and two external fields  $g_{ab}$  and  $\alpha$ . The transformation Eq.(2.10) preserve the value of this functional. In other words our effective  $(D-1)$ -dimensional Euclidean field theory is invariant under infinite-dimensional group parametrized by one function  $\Omega(x)$ . This transformation conformally modifies  $(D-1)$ -dimensional metric  $g_{ab}$ . In order to keep  $E$  invariant it is necessary to accompany it by additional transformation of the field  $\varphi$  and the dilaton  $\alpha$ . Let us emphasize that the equation Eq.(2.4) for the minimally coupled scalar field  $\Phi$  is not conformally invariant.

For point-like sources energy  $E$  diverges. To deal with this divergence one has to use some regularization and renormalization schemes. The regularized self-energy may not respect the invariance property in question. In quantum field theory the fact that renormalization procedure breaks some symmetries of the classical theory is the cause of appearance of conformal, chiral, etc. anomalies. In our case the same arguments are applicable to the renormalized self-energy of classical sources and their self-energy acquires anomalous terms. It has been shown [10] that the self-energy of point-like charges can be written in the form

$$E_{\text{ren}} = \alpha(x) \Delta m, \quad (2.13)$$

where

$$\Delta m = -\frac{q^2}{2} \mathcal{G}_{\text{reg}}(x, x) = -\frac{q^2}{2} \langle \varphi^2 \rangle_{\text{ren}}. \quad (2.14)$$

Here  $\mathcal{G}_{\text{reg}}(x, x)$  is the coincidence limit  $x' \rightarrow x$  of the regularized Green function

$$\mathcal{G}_{\text{reg}}(x, x') = \mathcal{G}(x, x') - \mathcal{G}_{\text{div}}(x, x'). \quad (2.15)$$

Here the Green function  $\mathcal{G}$  corresponds to the operator Eq.(2.8)

$$F \mathcal{G}(x, x') = -\delta^{D-1}(x, x'). \quad (2.16)$$

Thus, in order to find out the self-energy of a scalar charge one has to know the regularized Euclidean Green function corresponding to the operator Eq.(2.16). In the limit of coincident points this is exactly the  $\langle \varphi^2 \rangle_{\text{ren}}$  of a free scalar field. In other words the problem of calculation of  $\Delta m$  is formally equivalent to study of the quantum fluctuations  $\langle \varphi^2 \rangle$  in  $(D-1)$ -dimensional space. The only difference from our case is that in quantum field theory the amplitudes of vacuum fluctuations are normalized on  $\hbar^{1/2}$  while in the case of the self-energy of a classical charge they are normalized on unity. As for calculations it does not make any difference. There are many well-established methods of calculation of  $\langle \varphi^2 \rangle_{\text{ren}}$  in quantum field theory. All traditional methods of UV regularization like point-splitting, zeta-function and dimensional regularizations, proper time cutoff, Pauli-Villars and other approaches are applicable to this task. One can expect that all of them lead to the same predictions for the  $\langle \varphi^2 \rangle_{\text{ren}}$ .

Therefore, technically the problem of calculation of the self-energy of a charged particle reduces to the calculation of the quantum vacuum average value of  $\langle \varphi^2 \rangle_{\text{ren}}$ .

### III. CALCULATION OF $\langle \varphi^2 \rangle_{\text{ren}}$ AND ITS PROPERTIES

The Green function Eq.(2.16) transforms as follows

$$\mathcal{G}(x, x') = \Omega^{-\frac{n}{2}}(x) \bar{\mathcal{G}}(x, x') \Omega^{-\frac{n}{2}}(x'). \quad (3.1)$$

Therefore the non-renormalized value of  $\langle \varphi^2 \rangle$  should transform homogeneously

$$\langle \varphi^2(x) \rangle = \Omega^{-n}(x) \langle \bar{\varphi}^2(x) \rangle. \quad (3.2)$$

In other words the combination

$$g^{\frac{n}{2(n+2)}} \langle \varphi^2 \rangle \quad (3.3)$$

is an invariant under the transformations Eq.(2.10). This classical symmetry can be broken when one subtracts divergences from it, therefore

$$g^{\frac{n}{2(n+2)}} \langle \varphi^2 \rangle_{\text{ren}} \neq \text{const}. \quad (3.4)$$

However, one can find such anomalous term  $A(x)$  that restores the invariance property

$$g^{\frac{n}{2(n+2)}} (\langle \varphi^2 \rangle_{\text{ren}} + A) = \text{const}. \quad (3.5)$$

In order to derive this anomaly we use the Hadamard representation of the Green function. Consider the divergent part [20] of the Green function

$$\begin{aligned} \mathcal{G}_{\text{div}}(x, x') &= \Delta^{1/2}(x, x') \frac{1}{(2\pi)^{\frac{n}{2}+1}} \\ &\times \sum_{k=0}^{[n/2]} \frac{\Gamma(\frac{n}{2}-k)}{2^{k+1} \sigma^{\frac{n}{2}-k}} a_k(x, x'). \end{aligned} \quad (3.6)$$

When  $n$  is even the last term ( $k = n/2$ ) in the sum should be replaced by

$$\begin{aligned} &\frac{\Gamma(\frac{n}{2}-k)}{2^{k+1} \sigma^{\frac{n}{2}-k}} a_k(x, x') \Big|_{k=n/2} \\ &\rightarrow -\frac{\ln \sigma(x, x') + \gamma - \ln 2}{2^{\frac{n}{2}+1}} a_{n/2}(x, x'). \end{aligned} \quad (3.7)$$

Here  $a_k(x, x')$  are the Schwinger-DeWitt coefficients for the operator  $F$ . The world function  $\sigma(x, x')$  and Van Vleck-Morette determinant  $\Delta(x, x')$  are defined on the  $(n+2)$ -dimensional space with the metric  $g_{ab}$ . In order to extract the anomaly from the  $\mathcal{G}_{\text{div}}$  one has to know how the transformation Eq.(2.10) modifies  $\sigma$ ,  $\Delta$ , and  $a_k$  in the limit of  $x' \rightarrow x$ .

Here is a list of useful relations

$$\begin{aligned} g_{ab} &= \Omega^2 \bar{g}_{ab}, & g^{ab} &= \Omega^{-2} \bar{g}^{ab}, \\ g^{ab} \sigma_a \sigma_b &= 2\sigma, & \bar{g}^{ab} \bar{\sigma}_a \bar{\sigma}_b &= 2\bar{\sigma}, \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} \sigma_a &\equiv \sigma_{,a}, & \sigma_{ab} &\equiv \sigma_{;ab}, \\ \bar{\sigma}_a &\equiv \bar{\sigma}_{,a}, & \bar{\sigma}_{ab} &\equiv \bar{\sigma}_{;ab}. \end{aligned} \quad (3.9)$$

The notation  $()_{;}$  means the covariant derivative with respect to the metric  $g_{ab}$ , while  $()_{;}$  corresponds to the covariant derivative in the metric  $\bar{g}_{ab}$

$$\sigma_{ab} = g_{ab} - \frac{1}{3} \mathcal{R}_{acbd} \sigma^c \sigma^d + O(\sigma^{3/2}), \quad (3.10)$$

$$a_0(x, x') = 1, \quad (3.11)$$

$$\begin{aligned} \sigma &= \bar{\sigma} \left[ \Omega^2 - \Omega \Omega_{;a} \bar{\sigma}^a \right. \\ &\left. + \frac{1}{12} (4\Omega \Omega_{;ab} + 4\Omega_{;a} \Omega_{;b} - \bar{g}_{ab} \Omega_{;c} \Omega^{;c}) \bar{\sigma}^a \bar{\sigma}^b \right] + O(\sigma^{5/2}), \end{aligned} \quad (3.12)$$

$$\begin{aligned} \sigma &= \bar{\sigma} \Omega(x) \Omega(x') \left[ 1 + \frac{1}{12 \Omega^2} (-2\Omega \Omega_{;ab} \right. \\ &\left. + 4\Omega_{;a} \Omega_{;b} - \Omega_{;c} \Omega^{;c} \bar{g}_{ab}) \bar{\sigma}^a \bar{\sigma}^b \right] + O(\sigma^{5/2}). \end{aligned} \quad (3.13)$$

For the determinant  $\Delta^{1/2}(\mathbf{x}, \mathbf{x}')$  we have

$$\begin{aligned}\Delta^{1/2} &= 1 + \frac{1}{12} \mathcal{R}_{ab} \sigma^a \sigma^b + O(\sigma^{3/2}) \\ &= 1 + \frac{1}{12} \bar{\mathcal{R}}_{ab} \bar{\sigma}^a \bar{\sigma}^b \\ &\quad + \frac{1}{12\Omega^2} \left[ -n\Omega\Omega_{;ab} + 2n\Omega_{;a}\Omega_{;b} \right. \\ &\quad \left. - (\Omega\Omega_{;c}^c + (n-1)\Omega^{:c}\Omega_{;c}}) \bar{g}_{ab} \right] \bar{\sigma}^a \bar{\sigma}^b + O(\sigma^{3/2}).\end{aligned}\quad (3.14)$$

The difference of anomalous terms

$$B(x) = A(x) - \Omega^{-n} \bar{A}(x) \quad (3.15)$$

can be defined as

$$B(x) = \lim_{x' \rightarrow x} \left[ \mathcal{G}_{\text{div}}(x, x') - \frac{\bar{\mathcal{G}}_{\text{div}}(x, x')}{\Omega^{n/2}(x) \Omega^{n/2}(x')} \right]. \quad (3.16)$$

#### IV. CALCULATION OF A CLASSICAL SELF-ENERGY ANOMALY

To illustrate the described approach let us consider a couple of examples.

##### A. Four dimensions

In four dimensions  $D = 4, n = 1$

$$\mathcal{G}_{\text{div}}(x, x') = \frac{\Delta^{1/2}(x, x')}{4\pi} \frac{1}{(2\sigma)^{1/2}} a_0(x, x'). \quad (4.1)$$

Thus

$$\begin{aligned}\mathcal{G}_{\text{div}}(x, x') &= \frac{1}{4\pi} \frac{1}{(2\sigma)^{1/2}} + O(\sigma^{1/2}) \\ &= \frac{1}{4\pi} \frac{1}{(2\Omega(x)\bar{\sigma}\Omega(x'))^{1/2}} + O(\sigma^{1/2}), \\ \bar{\mathcal{G}}_{\text{div}}(x, x') &= \frac{1}{4\pi} \frac{1}{(2\bar{\sigma})^{1/2}} + O(\sigma^{1/2}),\end{aligned}\quad (4.2)$$

and, hence,

$$B(x) = 0. \quad (4.3)$$

Thus in four dimensions the anomaly vanishes and

$$\langle \varphi^2 \rangle_{\text{ren}} = \Omega^{-1} \langle \bar{\varphi}^2 \rangle_{\text{ren}}. \quad (4.4)$$

##### B. Five dimensions

In five dimensions  $D = 5, n = 2$

$$\begin{aligned}\mathcal{G}_{\text{div}}(x, x') &= \frac{\Delta^{1/2}(x, x')}{4\pi^2} \left[ \frac{1}{2\sigma} a_0(x, x') \right. \\ &\quad \left. - \frac{1}{4} (\ln \sigma + \gamma - \ln 2) a_1(x, x') \right].\end{aligned}\quad (4.5)$$

Taking into account that

$$\begin{aligned}\frac{\Delta^{1/2}}{\sigma} &= \frac{1}{\Omega\bar{\sigma}\Omega'} \left( 1 + \frac{1}{12} \bar{\mathcal{R}}_{ab} \bar{\sigma}^a \bar{\sigma}^b - \frac{1}{6} \Omega^{-1} \Omega_{;c}^{:c} \bar{\sigma}} \right), \\ \frac{\bar{\Delta}^{1/2}}{\bar{\sigma}} &= \frac{1}{\bar{\sigma}} \left( 1 + \frac{1}{12} \bar{\mathcal{R}}_{ab} \bar{\sigma}^a \bar{\sigma}^b \right),\end{aligned}\quad (4.6)$$

$$\mathcal{R} = \frac{1}{\Omega^2} (\bar{\mathcal{R}} - 6\Omega^{-1}\Omega_{;c}^{:c}), \quad (4.7)$$

$$V = \frac{1}{\Omega^2} (\bar{V} + \Omega^{-1}\Omega_{;c}^{:c}), \quad (4.8)$$

$$a_1 = \frac{1}{6} \mathcal{R} + V = \frac{1}{\Omega^2} \left( \frac{1}{6} \bar{\mathcal{R}} + \bar{V} \right) = \frac{1}{\Omega^2} \bar{a}_1. \quad (4.9)$$

Eventually we obtain

$$B = -\frac{1}{48\pi^2} \Omega^{-3} \Omega_{;c}^{:c} - \frac{1}{8\pi^2} \Omega^{-2} \ln(\Omega) \bar{a}_1 \quad (4.10)$$

and

$$\langle \varphi^2 \rangle_{\text{ren}} = \Omega^{-2} \langle \bar{\varphi}^2 \rangle_{\text{ren}} - B. \quad (4.11)$$

The anomalous contribution  $(-B)$  depends both on the physical metric and the reference one. It would be nice to get such expression for  $A$  which is a functional of only one metric and which gives Eq.(4.10) after its substitution to Eq.(3.15). One can easily see that

$$\begin{aligned}A(x) &= \frac{1}{288\pi^2} \mathcal{R} - \frac{1}{64\pi^2} \ln(g) a_1(x) \\ a_1(x) &= \frac{1}{6} \mathcal{R} + V\end{aligned}\quad (4.12)$$

is the wanted solution. Though the solution for  $A$  may not be uniquely defined, it is the functional  $B$  which is physically important and which is unambiguous.

##### C. Majumdar-Papapetrou spacetimes

In the paper [10] the self-energy of scalar charges has been studied in the higher dimensional Majumdar-Papapetrou spacetime [19]. This geometry is the solution of the Einstein-Maxwell equations which describes a set of extremely charged black holes in equilibrium in a higher dimensional asymptotically flat spacetime. The corresponding background metric and electric potential are

$$\begin{aligned}ds^2 &= -U^{-2} dt^2 + U^{2/n} \delta_{ab} dx^a dx^b, \\ A_\mu &= \sqrt{\frac{n+1}{2n}} U^{-1} \delta_\mu^0, \quad n = D - 3.\end{aligned}\quad (4.13)$$

Here the function  $U$  reads

$$U = 1 + \sum_k \frac{M_k}{\rho_k^n}, \quad \rho_k = |\mathbf{x} - \mathbf{x}_k|. \quad (4.14)$$

The index  $k = (1, \dots, N)$  enumerates the extremal black holes.  $x_k^a$  is the spatial position of the  $k$ -th extremal black hole.

One can see that the transformation Eq.(2.10) with

$$\Omega(x) = U^{1/n}(x) \quad (4.15)$$

connects the Majumdar-Papapetrou metric Eq.(4.13) to the Minkowski  $D$ -dimensional metric, that is  $(D - 1)$ -dimensional flat space  $\bar{g}_{ab}$  and simultaneously  $\bar{\alpha} = 1$ . Because in flat spacetime with  $\bar{V} = 0$  the quantity  $\langle \varphi^2 \rangle_{\text{ren}} = 0$  the invariance property Eq.(3.5) automatically gives

$$\langle \varphi^2 \rangle_{\text{ren}} = -B. \quad (4.16)$$

Then the renormalized self-energy of a scalar charge can be obtained from Eqs.(2.13)-(2.14).

In four dimensions our approach trivially gives

$$\Delta m = 0. \quad (4.17)$$

In five dimensional Majumdar-Papapetrou spacetime,  $a_1 = 0$  and we reproduce the result [10]

$$\Delta m = \frac{q^2}{576\pi^2} \mathcal{R}. \quad (4.18)$$

where  $\mathcal{R}$  is the Ricci scalar of the spatial metric  $g_{ab}$ .

The analysis of the structure of divergent terms of the scalar Green function leads to the conclusion that the anomaly in question should vanish in all even-dimensional spacetimes.

## V. CONCLUSIONS

In this paper we have presented an approach to study the self-energy of pointlike charges based on calculation of the self-energy anomaly. Our approach is applicable to arbitrary static spacetimes. The self-energy

of static scalar sources of a minimally coupled massless scalar field is invariant under special symmetry transformations Eq.(2.10). These local transformations consist of simultaneous multiplication of the  $g_{tt}$  component of the metric by a scalar function of spatial coordinates and conformal transformation of the spatial part  $g_{ab}$  of the spacetime metric with the conformal factor being some power of the same function. In the case of Majumdar-Papapetrou spacetimes it happens that this symmetry relates Majumdar-Papapetrou spacetimes to the flat Minkowski spacetime. Therefore the calculation of the self-energy can be reduced to its calculation in a flat metric, that is trivial. The subtle point is that regularization of the UV divergent quantities violates the invariance property of the classical energy of the charge distribution and leads to an anomalous contribution to the renormalized self-energy. In Majumdar-Papapetrou spacetimes the exact Green functions are known [20] and straightforward calculations [10] showed that this anomaly, in fact, constitutes the whole effect. This exact transformation law makes possible to relate the self-energy of a charge in the physical spacetime to the self-energy in some reference spacetime, where its calculation may be significantly simpler. The proposed approach may provide one with tools for construction of approximate methods of calculation of  $\Delta m$ , e.g., similar to the Page approximation [21] developed for calculation of quantum vacuum fluctuations  $\langle \Phi^2 \rangle_{\text{ren}}$  and of the stress-energy tensor of fields in static spacetimes.

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